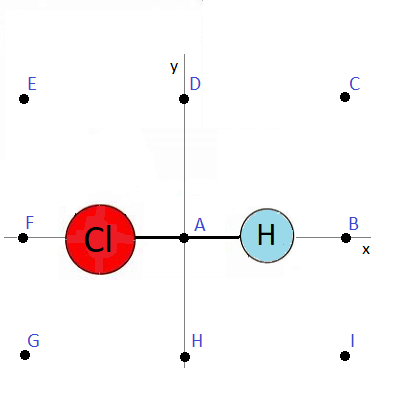
***Homework 1: Electric Fields (Solutions)***

\* for the problems that require integration, my main concern is that you’re able to set the integral up correctly. But you needn’t then *explicitly* calculate the integral yourself (by finding the anti-derivative, etc.). You can just plug it into your calculator, Wolfram Alpha, or whatever.

**Problem 1.** A Hydrogen Chloride molecule is centered at the origin. It has a bond length of 127pm. Cl has 7 electrons in its valence shell, whereas H has 1. To complete its valence shell, Cl tends to ‘borrow’ H’s electron, giving it an ‘effective’ charge of -1e, and leaving H with an ‘effective’ charge of +1e, where e = 1.6×10-19C. For this molecule, calculate (and draw) the magnitude and direction of the electric field at the 9 points below. That’s a lot of points, and I partially hate myself for assigning it; but note that once you’ve calculated some of them, the rest may be obtained by symmetry.



(a) Point A = (0pm, 0pm).

So,



(b) Point B = (100pm, 0pm)?

And,



(c) Point C = (100pm, 100pm)?

Yep,



(d) Point D = (0pm, 100pm)?

So….



(e) Point E = (-100pm, 100pm)?

Could get this by symmetry, but I’ll do it out….



(f) Point F = (-100pm, 0pm)?

Could get this by symmetry too, but I’ll do it out,



(g) Point G = (-100pm,-100pm)?

Again, could do symmetry, but:



(h) Point H = (0pm, -100pm)?

Symmetry, or….

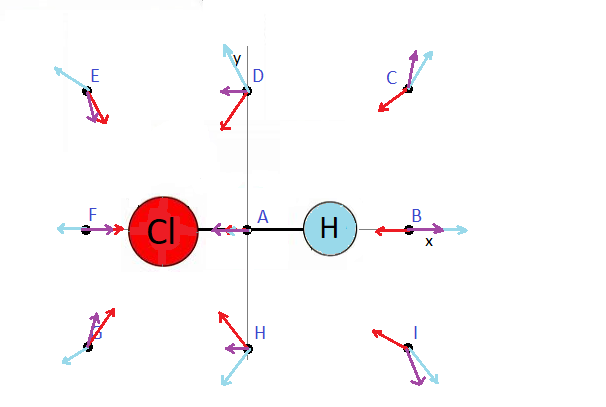


(i) Point I = (100pm, -100pm)?

Symmetry, or….

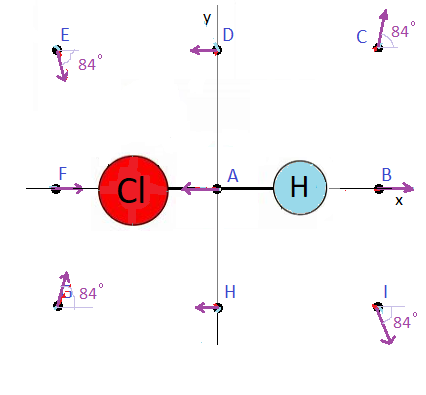


Plot looks like (net field in purple),

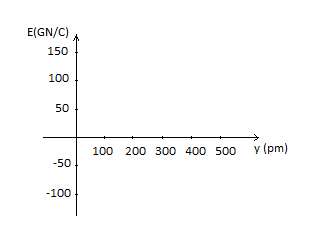
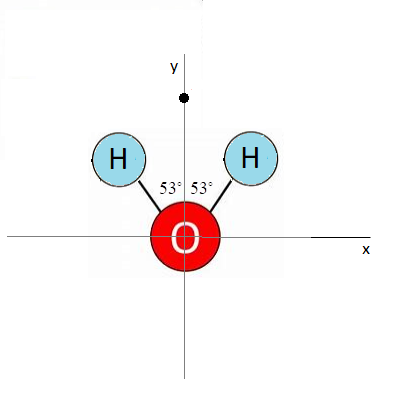


And to summarize,





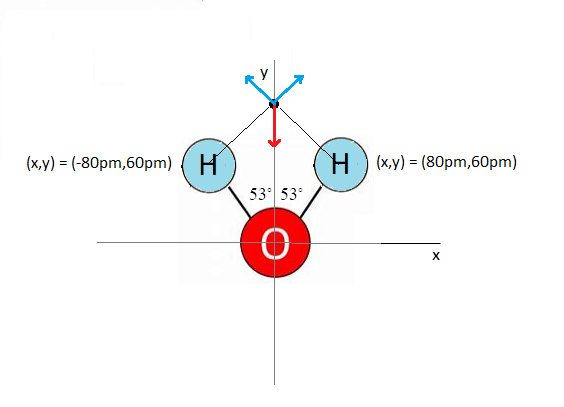
**Problem 2.** A water molecule is situated with the O-atom at the origin, and the two H-atoms centered a distance 100pm away from the origin, making a 53° angle with the axis of symmetry (y-axis). O has 6 electrons in it’s outer shell, whereas H as 1. To form a complete shell (8 electrons), O partially ‘borrows’ the H’s electrons so that they spend more of their time near O. This results in O having an ‘effective’ negative charge of qo = -0.70e, and the H’s each having an effective negative charge qH = 0.35e, where e = 1.6×10-19C.



(a) Write down an expression for the electric field generated by the water molecule, along the +y-axis. And draw a rough plot (can use your calculator, someone else’s calculator, stolen calculator, free online graphing software, etc.) of the field below. Note GN/C = 109N/C, and pm = 10-12m.

So, first we need the coordinates of the two H-atoms. The right H-atom’s coordinate is: (100pm×sin(53°), 100pm×cos(53°)) = (80pm, 60pm). And the left H-atom’s coordinate would be:

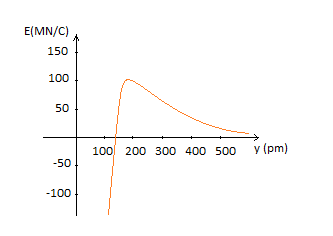
(-80pm, 60pm).



Next, we just add up all the fields:



Plot gives something like this:



(b) Between what range of y’s is the field pointing up?

It’s pointing up between about (131pm, ∞).

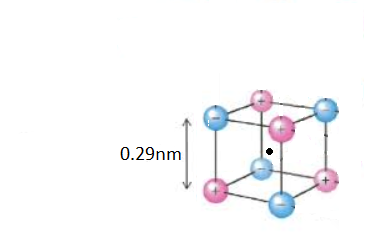
(c) Between what range of y’s is the field pointing down?

It’s pointing down between about (0pm, 131pm).

(d) Where is the field zero? Side note: this electric field is what exerts the so-called Van der Waals force. It encourages neighboring molecules to attract each other, and settle into the point where **E** = 0, and thereby form a liquid state. This will happen when the temperature is low enough (or pressure great enough).

It’s zero at around y = 131pm.

**Problem 3.**  Consider a single cubic section of a sodium chloride crystal. Each Na (pink) atom donates and electron to a Cl atom (blue), leaving the Na atom with a charge of +1e, and each Cl atom with a charge of -1e. Might want to think about it before you jump into a calculation.



(a) What is the electric field at the center of the crystal (black dot)

It’s zero, by symmetry.

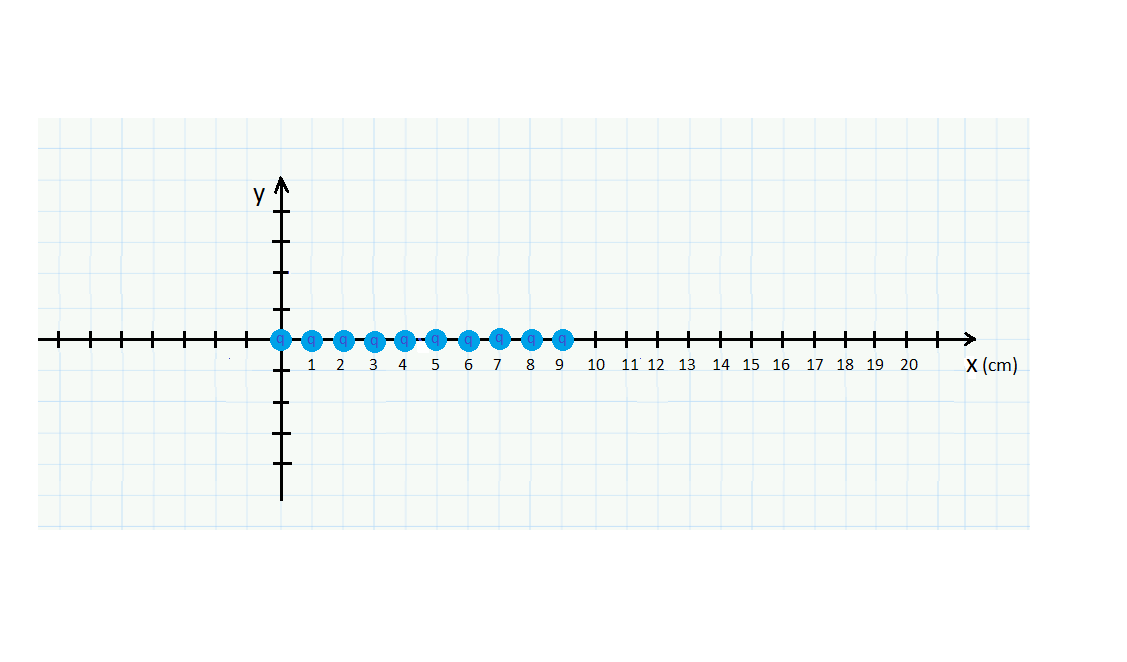
(b) Suppose we move the black dot upwards 0.1nm from the center. What is the electric field there?

Still zero.

(c) Now move the black dot upwards an additional distance of 0.045nm to the center of the top face of the crystal. What is the electric field there?

zeeeeero.

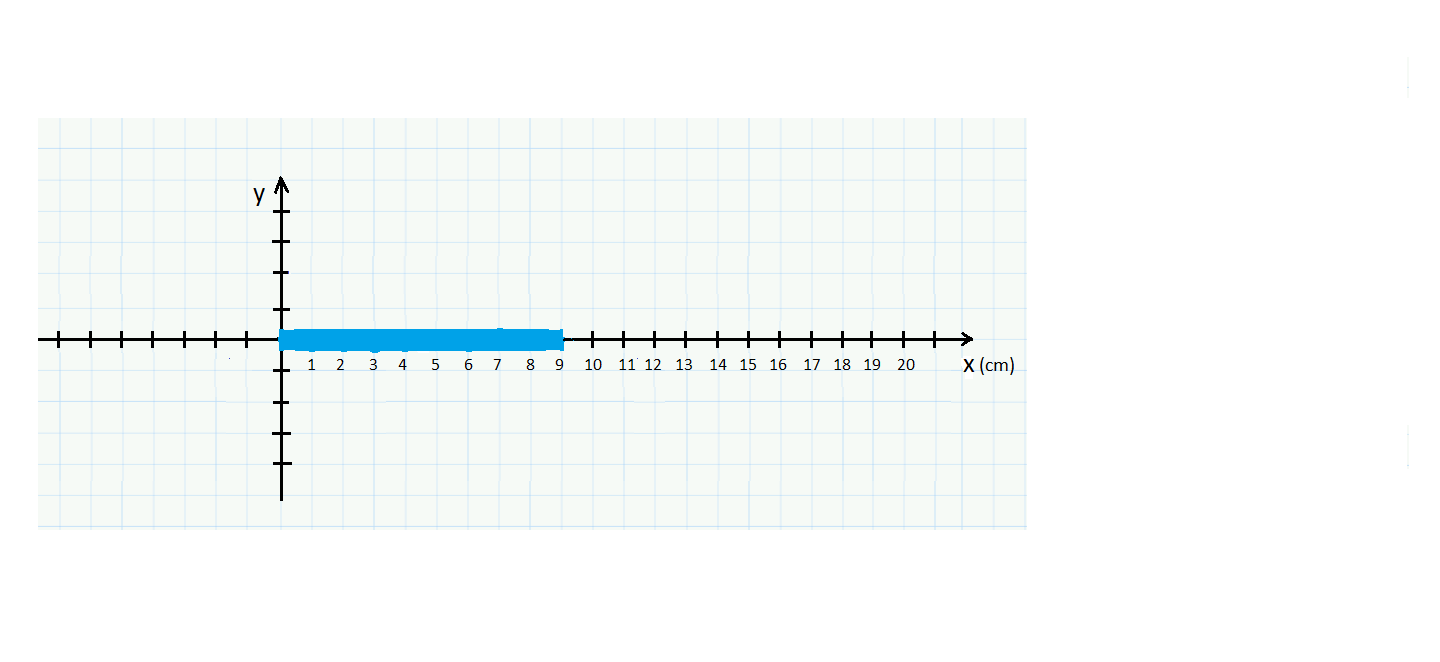
**Problem 4.** Consider ten 1nC charges arranged in 1cm intervals along the x-axis. What is the electric field at x = 20cm? Note you might want to algebraically simplify your expression a little before embarking upon the calculation.



Field is:

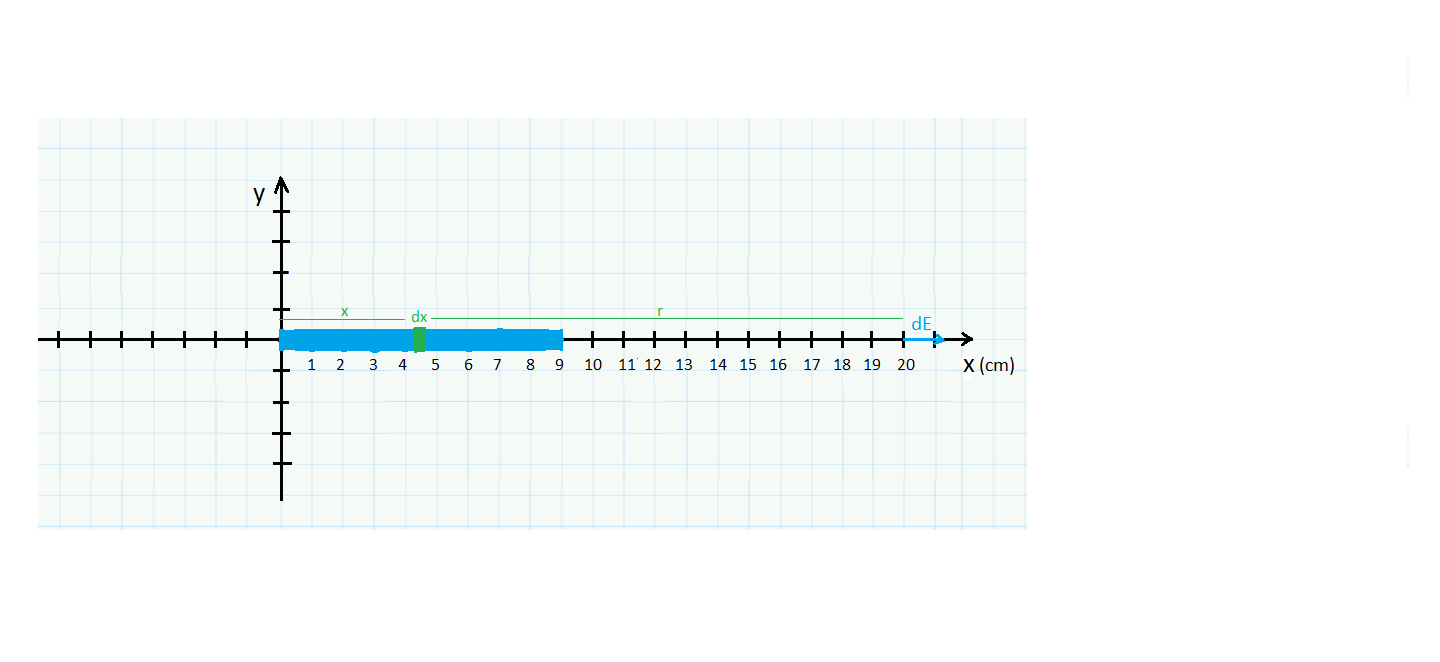


**Problem 5.**  Now reconsider *problem 4*, but take the charges and smear them continuously over the (0cm, 9cm) interval.



(a) What is the electric field at x = 20cm? How does this answer compare to *problem 4*? The main point of this exercise to illustrate that as long as the charges are close together, it’s a pretty good approximation to smear them together, which affords a much quicker calculation, than doing it one by one, especially if there are not 10 particles, but 1023 particles, as is more typical.

So if we single out a piece of charge dq, and calculate its field dE, and then integrate over all dE’s we’d get:

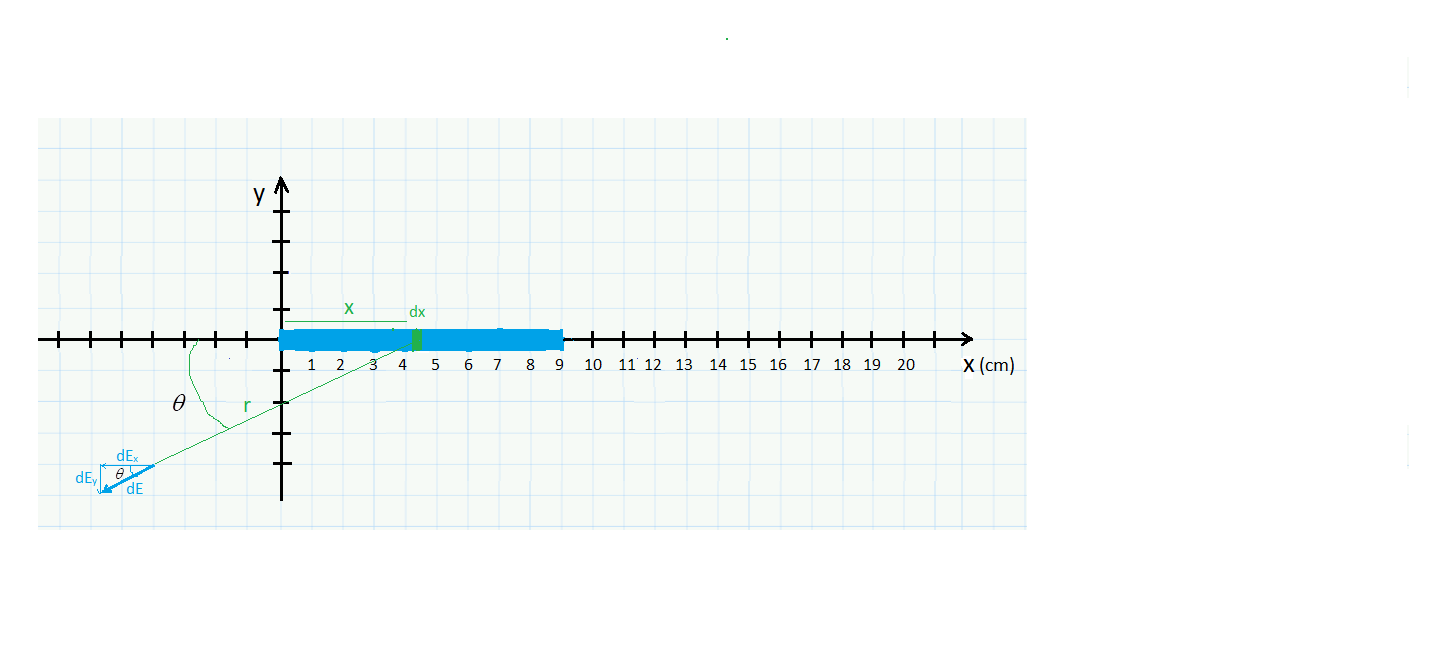




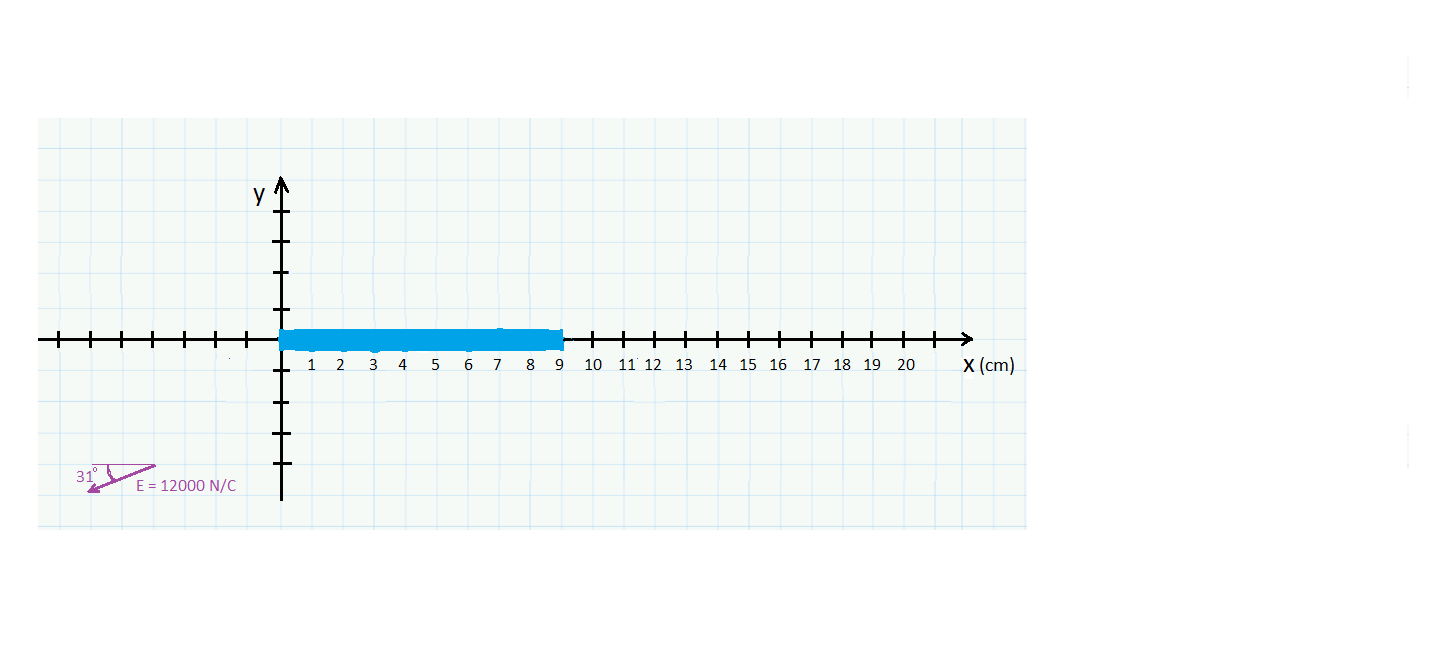
It’s pretty close ;).

(b) Now calculate (and draw) the magnitude and direction of the field at the coordinate (-4cm, -4cm).

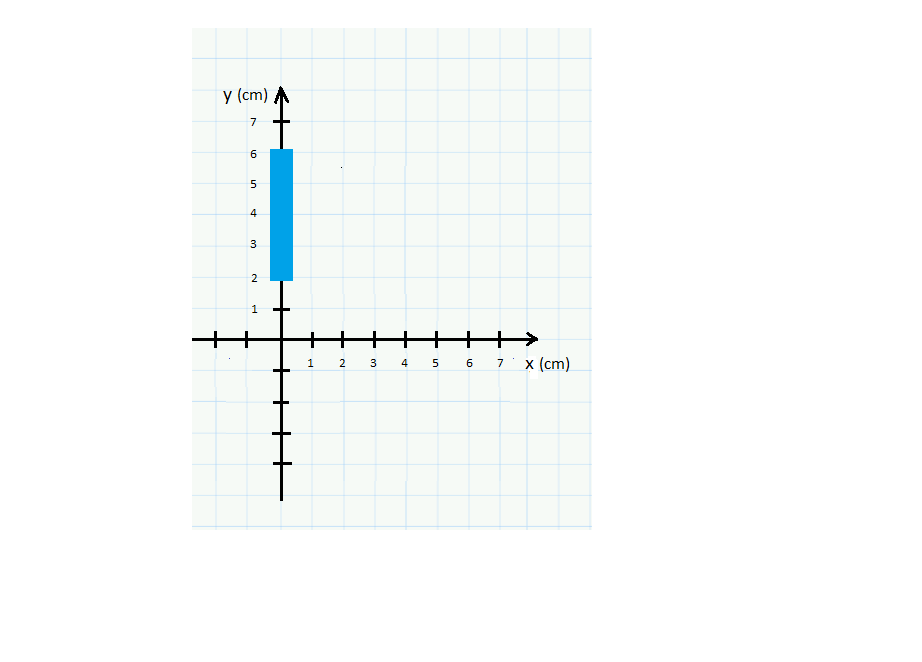
So if we single out a piece of charge dq, and calculate its field dE, and then integrate over all dE’s we’d get:





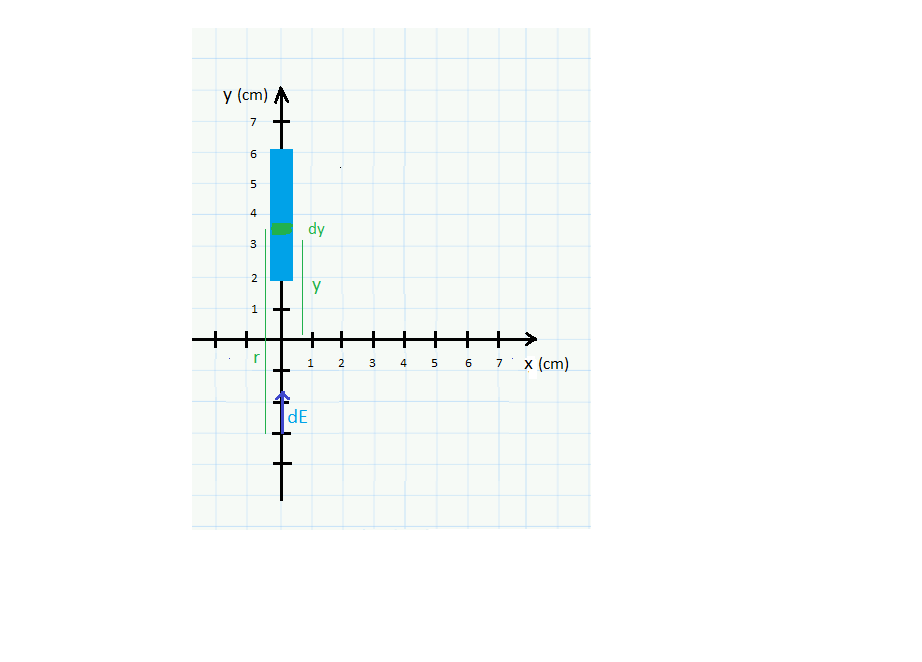


**Problem 6.** Consider a plastic rod charged non-uniformly as λ(y) = -2y (nC/m).

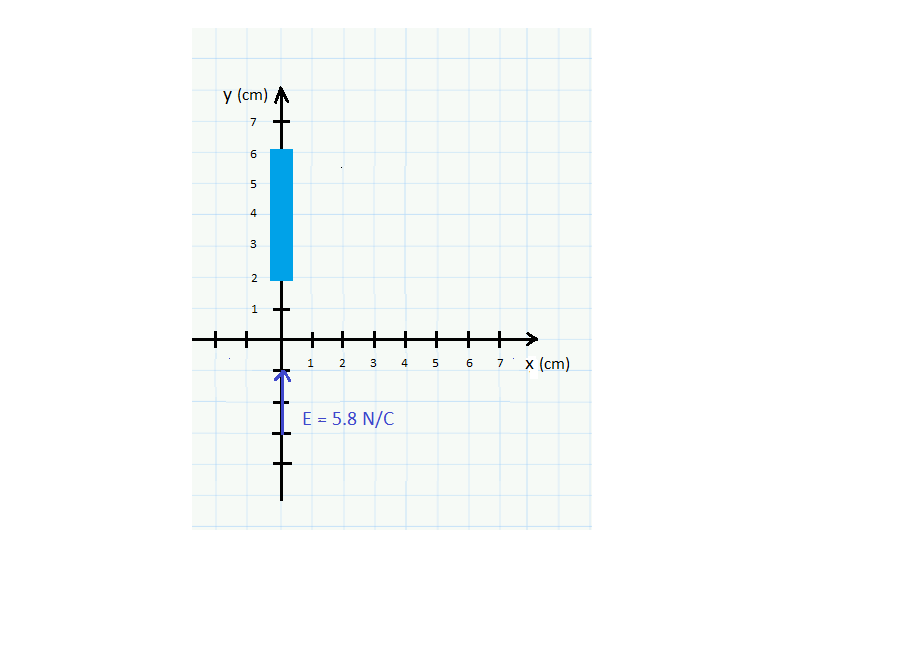


(a) Calculate (and draw) the magnitude and direction of the electric field at the point y = -3cm.

So if we single out a piece of charge dq, and calculate its field dE, and then integrate over all dE’s we’d get:

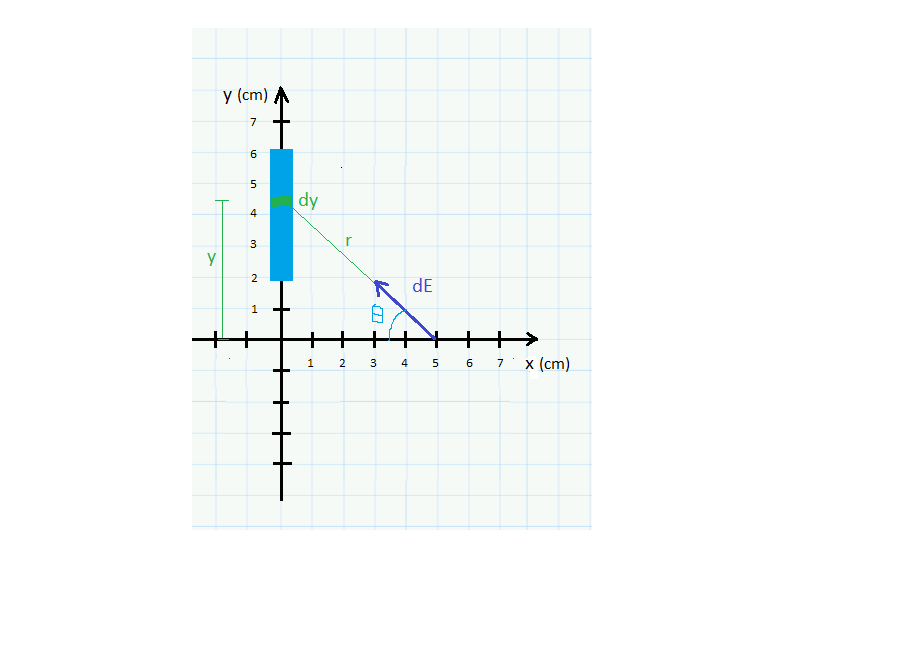




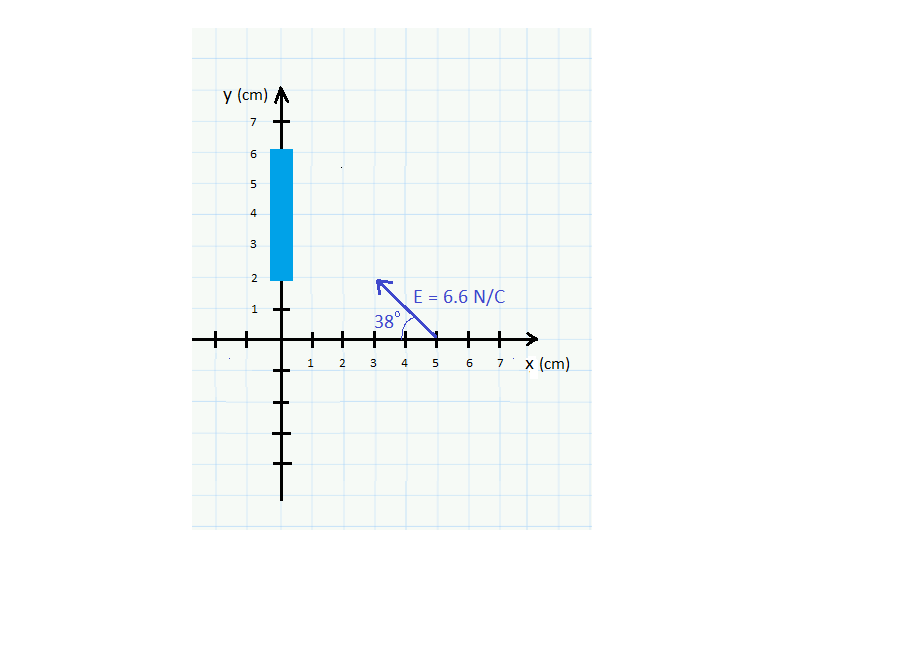


(b) Calculate (and draw) the magnitude and direction of the field at x = 5cm on the x-axis.

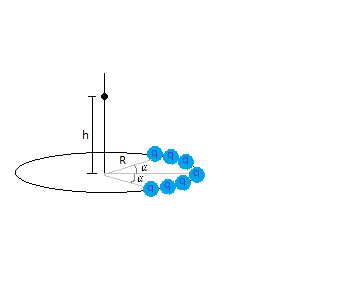
So if we single out a piece of charge dq, and calculate its field dE, and then integrate over all dE’s we’d get:



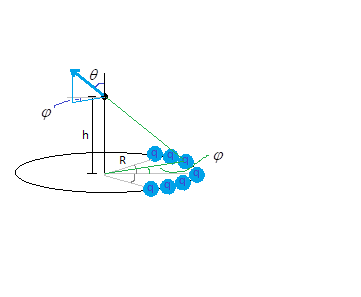




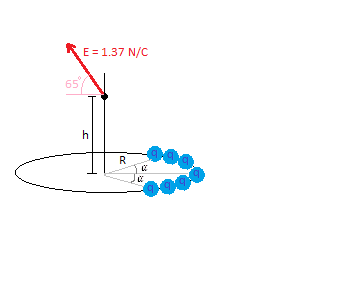
**Problem 7.** Consider seven q = 1nC charges arranged between α = ±30° in 10° intervals along the circumference of the R = 3m radius circle as shown. Calculate (and draw) the magnitude and direction of the electric field at the point h = 2R on the z-axis. Note you might want to algebraically simplify your expression a little before embarking on the calculation; i.e., leave φ as a variable, simplify as much as possible, and then add up all the fields, substituting the φ for each one. This will be faster than calculating the fields one by one.



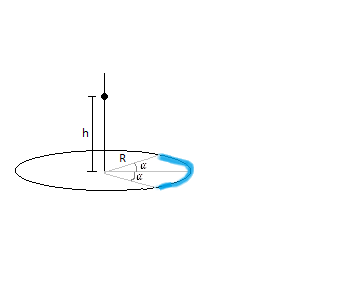
We’ll consider an arbitrary charge at angle φ from +x axis, construct its field, and then add them all up.



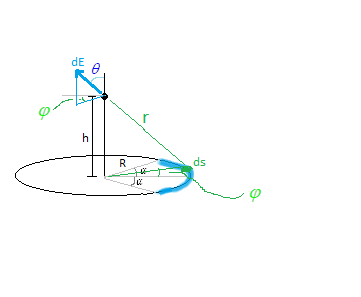




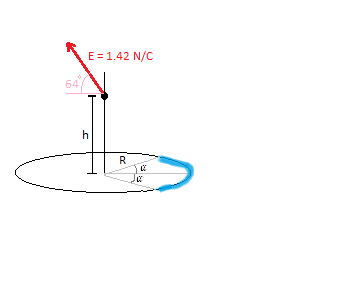
**Problem 8.**  Now reconsider *problem 7*, but this time let’s smear the 7nC charge evenly over the α = ±30° interval. Recalculate (and draw) the magnitude and direction of the field at point h. How does it compare to problem 7’s result? The point of this exercise is to illustrate that the continuum approximation is a good one, when charges are close together.



So if we single out a piece of charge dq, and calculate its field dE, and then integrate over all dE’s we’d get:

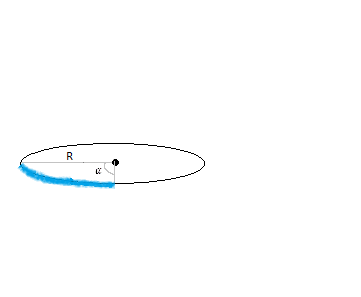




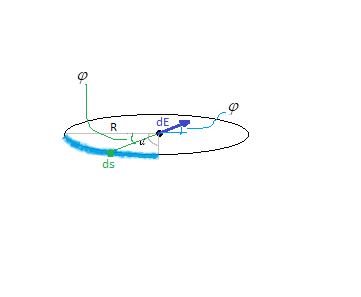


Pretty close, I say.

**Problem 9.** Let’s consider a charged semi-ring one more time. Suppose we change the charge density so that it varies with angle: λ(φ) = 2φ (nC/m). What is the field at the center of the ring? α = 90° and R = 3m.



So if we single out a piece of charge dq, and calculate its field dE, and then integrate over all dE’s we’d get:





which looks like this:

